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IMAGE ANALYSIS IN PAPER MANUFACTURING*

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Abstract

Significant energy savings in the paper industry can result from characterization of paper pulps by direct measurement of fiber morphology with automatic methods. We discuss the various fiber properties for which automatic image analysis applied to microscope images of pulps can yield savings of materials and energy. Computational algorithms applied to actual fiber data are shown to yield useful measurements for fiber length, curl, and potentially other properties. New fiber morphology measurement algorithms can include curl and length as special cases. Potentials for collaboration between the paper industry and the image analysis instrument industry are explored.

Keywords: algorithms; fiber morphology; fibers; image analysis; pulp characterization; pulps; pattern recognition.

I. Measurement Problems in Paper Manufacturing

The paper industry, which is the fourth largest consumer of energy in the United States today, uses large quantities of raw materials to produce the various grades of paper and other products from pulps. Economical paper production depends on reliable characterization of these pulp materials.

Historically, this characterization has been achieved by using materials which have constant characteristics from one source of supply to the next--typically, virgin pulps from well-controlled and well-understood timber supplies. But even the use of virgin pulp requires characterization before it can be placed into production use.

One reason why the recovery of waste paper products as an alternative source of supply has not been heavily practiced is because the difficulty of predicting the properties of raw materials coming from heterogeneous sources places an insurmountable burden on the characterization problem.

Paradoxically, the methods for characterizing raw pulp materials are well understood in the sense that there are manual methods for determining many pulp properties that predict the properties of the resulting manufactured product. Most of these methods have not received the study necessary to lead to any degree of automation.

It is the purpose of this paper to consider methods for automating the measurement of pulp properties, especially those of a morphological nature that can be used to predict the properties of the resulting paper product.

Pulp fibers, especially chemical pulp fibers, are not suitable for paper manufacture as produced by a pulp mill. Paper made from untreated pulp is very weak and porous, has a low density, and the mass distribution is very disperse. In order to make paper, having suitable properties, it is necessary to subject the fibers to mechanical action. Mechanical action, such as beating or refining, results in changes in the morphological structure of the fibers essential to making good paper.

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During the beating or refining process the fibers imbibe water and swell, fibrils form and extend from the fiber wall, fibers are cut, surface area increases, debris is produced, the fibers become more flexible, the outer walls of the fiber are removed and internal bonds are broken.

The degree and type of change in the morphological structure of fibers resulting from mechanical action depends on the design of the mechanical equipment, the type of pulp used, the energy imparted to the fibers, the temperature, consistency of the pulp slurry, duration and many other variables. It is desirable to control the variables in mechanical refining consistently to produce the desired type and degree of morphological change in the fibers and at a minimum expenditure of energy. But in general, great reliance is placed on prior experience for adjustments of the many variables inherent in the process. In essence the mechanical action process has no reliable on-line quality control measurement techniques available to it.

Mechanical pulps derive their name from the fact that wood is disintegrated into a fibrous state mechanically as opposed to chemical pulps which are produced by the dissolution and removal of lignin which holds the fibers together causing the wood to disintegrate into its component fibers. It is to be recognized that mechanical pulps contain practically all of the lignin of the original wood. When one considers that pulp yields from mechanical pulping are as high as 95 percent and that this pulping process generates little air or water pollution, it is easy to understand why the paper industry is turning more and more towards mechanical pulps for their source of virgin fiber.

The advantage in using mechanical pulps arose from the economic advantages stemming from the high yield of pulp and cheap hydroeléctric power. However, during the past several years cheap hydroelectric power has vanished. The price differential between chemical and mechanical pulps is not as great as it once was. Any development which would lead to a substantial savings in energy consumption in the mechanical pulping industry would be extremely beneficial.

In making mechanical pulp, as in any other pulping process, it is imperative that a usable, uniform pulp be consistently produced at minimum costs to make a satisfactory product. Once the qualities needed in a mechanical pulp have been established it is essential that those qualities be maintained. Consequently, quality control tests are needed which will accurately reflect the quality of the pulp. To complicate the situation, ground wood, which is the mechanical pulp most frequently manufactured, is a mixture of pulps from a number of grinders each of which produces a pulp of a different quality. Furthermore there are many different grades of ground wood pulp and until recently there have been no simple quality control tests to cover all the various grades of ground wood pulp.

In 1963, Forgacs[1] showed that mechanical pulps could be characterized by the weight-average fiber length and by a shape factor which was related to the hydrodynamic specific surface of the fibers. This characterization was based on a measure of weighted fiber length, a measurement of the adhesive quality of the fines, and a measurement of the number of shives. Shives are bundles of incompletely separated fibers which cause imperfections in paper. The long fiber component of the pulp serves to strengthen the paper whereas the fines provide the adhesive. Measurement of the adhesive quality of the fines requires a knowledge of the relationship between fiber length and specific surface in the pulp.

Although the measurements necessary for the characterization of mechanical pulps have been identified, no on-line system for these measurements has been developed. As late as 1967, it was believed that it would be impractical to make measurements of fiber length and surface area on a routine basis because of the tediousness and duration of the measurements.[2] The approach taken was to make indirect measurements of the properties to be measured. A system was designed for the on-line measurement of the properties which gave an indirect measurement of the critical parameters. The system appeared to perform satisfactorily under mill conditions. However, apparently it was not sufficiently sensitive to resolve significant differences in pulp quality.

In recent years a new mechanical pulping process, thermomechanical pulping, has gained in popularity. Use of thermomechanical pulp (TMP) in newsprint permits the elimination of chemical pulp.[3] This can mean an overall fiber savings of more than 15 percent. More importantly the manufacturing of newsprint would rely less on chemical pulps which originate from more polluting pulping processes. In addition it has been possible to reduce the total amount of fiber used to produce certain types of paper without sacrificing quality, thus giving rise to material savings.

Even though TMP has so many advantages, pulp has not been produced with TMP consistent with the desirable properties it potentially can have, primarily because there are no on-line measurement techniques which will allow for the control of manufacturing variables which effect pulp quality. As a consequence, newsprint and other publication grades of paper manufactured from TMP still rely on chemical pulps to insure meeting of performance specifications for the paper. In many instances paper is over-engineered in order to assure that the paper produced can at least meet specifications consistently. If the specifications are not met, the paper has to be merchandised at a lower grade or reprocessed. Neither alternative is acceptable.

TMP is also perhaps the most energy intensive of all the mechanical pulping processes. The greater utilization of energy is, in large part, owing to the necessity of consistently assuring a quality pulp. Without a doubt an on-line measurement system is essential for TMP if this method is consistently to produce high quality pulps with a minimum amount of energy. Fiber length distribution and shape factors are the parameters essential to the quality determination of TMP pulps.

Up to now the only morphological properties we have discussed were fiber length, geometric area and shape. There are other morphological characteristics which are equally and in some cases more important. One such characteristic is fiber curl. Pulp fibers are usually bent to varying degrees during pulp manufacturing. This bending is termed "curl". The curliness of fibers has a marked effect on the mechanical properties of paper, especially its extensibility. Also, curly fibers will not respond to stresses as do straight fibers. In some instances a high degree of curl is desired while in others it is undesirable. While the effect of curl is known[4] there is no quantitative method available for the measurement of curl.

Another important morphological characteristic of pulp fibers is coarseness which is expressed as the weight per unit length of fiber. The coarseness of fibers has a great effect on strength, smoothness and folding endurance as an example. Coarseness is dependent on fiber thickness, the size of the central canal or lumen of the fiber and the density of the cell wall material. Because of the low total weight of the samples currently used to determine coarseness, and because of the natural variation of fibers, errors in coarseness measurement are quite possible. One means of reducing the error is to increase the sample size but then the measurement time also increases.

In addition to the above morphological characteristics, measurement of fiber slenderness, fiber defects, determination of whole versus fractional fibers, flatness, classification as to source, and classification as to tree species are also properties of importance. Automation of their measurement would be of great importance to the paper industry.

Consider the problems in using pulps made from waste paper consisting of a mixture of paper grades. Mixed waste paper varies in composition from source to source and from day to day from a single source. Because of this great variation and unknown composition mixed waste is used only for the lowest of paper grades and consequently has the lowest utility of all waste paper grades. Unfortunately, mixed waste is the most abundant grade of waste paper available. If paper recycling is to increase significantly, greater utilization of this grade of waste paper must occur.

Paper stock dealers are usually uncertain of the long range demand for mixed waste paper, resulting in great reluctance to handle this grade of paper stock. Processors are uncertain of the ability of suppliers to provide an acceptable mixed waste over an extended period of time. Furthermore, it is always questionable whether the equipment to process mixed waste paper is sufficiently reliable to produce a product which will meet the customers' specifications.

Some of the uncertainties could be removed by developing analytical and physical test methods which would enable either the waste paper or the pulp produced from the waste paper to be characterized. The great variability that is so common with mixed papers could then be coped with by paper manufacturers and there would be a greater probability of producing products capable of meeting customer specifications. Appropriate test methods could thus provide a greater incentive for recycling by reducing many of the economic and technological uncertainties.

The greatest concerns of paper recycling are the morphological and mechanical properties of the pulp fibers and the cleanliness of the pulp. Clean pulps are necessary for good performance on paper machines and for appearance properties. The importance of the morphological and mechanical properties of pulp fibers have been discussed earlier.

At first glance, it would appear that the requirements for characterizing recycled pulps is not very different from virgin pulps with the exception of dirt measurement. In a sense this is the case. However, variations in mixed waste paper pulp quality occur during processing because of the inherently variable material. It is necessary constantly to monitor the quality of recycled pulps so that as variations in quality arise, the pulp can be directed to an area corresponding with the quality of pulp being produced.

II. The Technology of Fiber Morphology Measurement

Microscopy is an established method for measuring the morphological characteristics of fibers. The techniques are well developed and extensively used. But because of the tediousness and duration of the measurement, microscopy is almost never used for quality control. What we are addressing is the question of automating the microscopic measurement techniques for fiber morphology, decreasing the duration and tediousness of the measurements and, in the process, making microscopy promising for quality control of the mechanical action processes so important in the paper manufacturing industry. It would appear that image analysis and pattern recognition techniques are suited for this purpose. They could be used to develop an on-line measurement system for the quality control of mechanical pulp. Fiber length distribution could be determined by image analysis and expressed in whatever form is most suitable. It may be possible to obtain a measure of the specific area of a pulp by measuring the geometric area of the pulp fibers. And it should be entirely possible to characterize the shape factors for the pulp as well. Image analysis technology certainly appears to have all the necessary attributes for an on-line system for characterizing mechanical pulps.

For such an on-line system, we will argue below by demonstration that some of these characterizing measurements can be made with existing image analysis technology. It remains here to discuss the general requirements that must be met before such technology can be brought to bear on the particular measurement problems we are discussing.

Image analysis technology draws upon contributions from optical engineering and from digital computer developments as well as from recent advances in microelectronics. It has had notable successes in such areas as quantitative analysis of metallurgical preparations, routine analysis of such biological preparations as blood cells, the analysis of x-ray photographs, and the analysis of images obtained from satellites in monitoring earth resources. The technology is, of course, comparatively recent in its mature development; its earliest accomplishments only go back as far as 1957.[5]

But certain common characteristics have consistently been evident in all successful applications of image analysis technology. In all cases there is need for a transducer for capturing the basic optical data whether it be data from an x-ray detector, earth reflectivity to a satellite, or microscope optical data in a blood cell analyzer. The scanning transducer device serves two functions: first, that of gathering the basic data and second, in particular cases, the additional function of being able to be directed to a special point of interest in a source of data otherwise too complex or too large for purposes of complete analysis. Thus in the analysis of images obtained from pulps one would anticipate the need for a device which could scan a microscope field of fibers in a dilute suspension in order to find those further to be analyzed.

A second major characteristic of image analysis instruments is the internal digital logic which governs the analysis that they perform. This digital logic assumes two forms in most of the developments to date. The first and earliest form is that of the general purpose programmable computer in which the program, written independently of the computer and fed into it, governs the analysis made on the scanned data. The other form of the device is functionally similar to the first but does not have programming capability. Rather it takes the program from an otherwise programmable machine and reduces it usually to microprocessor hardwired logic with resulting increases in speed and decreases in flexibility. Both the programmable and the hardwired types of devices are capable of performing a wide variety of measurements. What determines the type of device suitable in any one particular image analysis instrument is the precision and completeness with which the kind of analysis to be performed can be specified beforehand. It is axiomatic in the computer field that exhaustiveness, completeness, and ultimate precision are requirements that must be met by any procedure which is to be automated. Only occasionally is it possible to design an analysis procedure which by default can invoke human assistance in order to supplement an otherwise incomplete algorithmic description. A main purpose of our presentation here is to suggest some algorithms that meet these desiderata as candidates for possible full automation of paper fiber analysis.

The kind of stringent requirements that must be met by image analysis algorithms before they can serve as the basis for useful instruments have typically been achieved in either of two ways. In the first way, experimental developments using programmable computers have led in an evolutionary fashion to specifications for instruments which carry out the algorithms that have been developed on the general purpose machines. A second, more recent approach[6] involves successful developments in which existing instruments that performed laboratory analyses were automated by introoducing microprocessor based computer control. It would be fortunate to be able to start with existing image analysis instruments currently used for paper fiber analysis and to suggest methods of automating these existing instruments. Unfortunately, the type of measurements we are concerned with here are not currently subject to any degree of routine automatic or semiautomatic measurement. Therefore, the more evolutionary approach recommends itself, in which general purpose programmable instruments are used to test measurement algorithms. To the extent that these algorithms prove successful in prototype, they can be reduced to hardware implementation with specialized microprocessor based technology. Although this evolutionary approach is certainly more time consuming, the current pace at which such developments are reduced to implementation in high speed microprocessors leads one to be optimistic about the development times involved. The approach we take, therefore, with respect to automating the measurement of morphological properties for paper fibers is the evolutionary one in which we develop measurement algorithms on a general purpose programmable computer and then use these algorithms as specifications for subsequent instrument developments which can carry out the same algorithms but less expensively and very much faster.

III. Fiber Measurement Algorithms

A. Semiautomatic Data Acquisition

For purposes of defining an image analysis algorithm and exhibiting the results of such an analysis, we adopt here the procedure of using semiautomatic methods for data acquisition. We will discuss subsequently the methods for acquiring these data with image scanners. The data to be analyzed consists of a set of photomicrographs such as that shown in Figure 1 in which a separable but nevertheless partially overlapping set of fibers appears. Manual measurements on these fibers can be made by using graphic stylus techniques. We have written programs which use a graphic stylus to trace the fibers in Figure 1 and to provide to the computer coordinate information concerning the location of points traced along each of the fibers.

With a 10.2 centimeter by 12.7 centimeter print photomicrograph and a graphic tablet having a resolution of 0.25 millimeters, we can manually trace each of the fibers by successively placing the graphic stylus on points arbitrarily chosen along the length of the fiber and automatically register the x and y coordinates

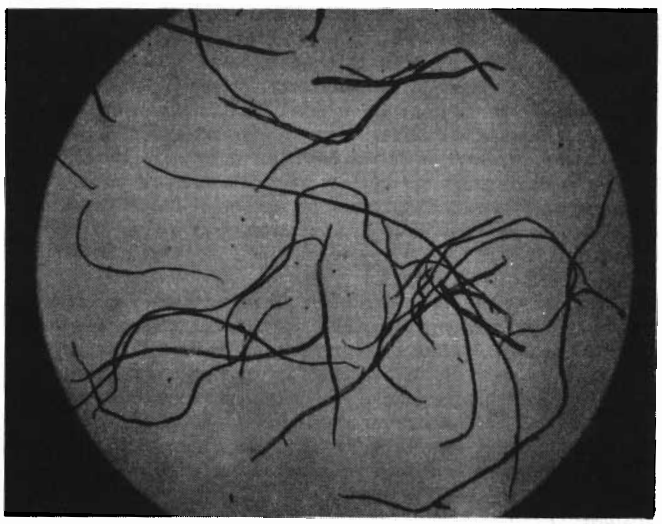

Figure 1: Photomicrograph of a Southern Softwood Pulp Showing Individual Fibers

of such points. With the programs we use it is possible to tell the computer which points constitute the beginning and end of each fiber. This manually solves an important image analysis problem of separating individual fibers which overlap. *In Figure 2 we show* a display produced by the computer exhibiting for each internal point measured along each fiber the location of that particular point. Only those fibers lying wholly within the microscope field have been traced.

This method of manually tracing fiber photomicrographs inherently contains certain errors. There is the error associated with imprecise placement of the graphic stylus on the image. There is also an error associated with the nonsystematic choice of measurement points along the length of the fiber. For the data that was analyzed in this experiment, points were chosen according to a nonrepeatable criterion in which more points were plotted in regions where fibers bend and fewer in straight sections of such fibers as can be seen in Figure 2.

Notice that with this semiautomatic method of acquiring the image data, it is possible by straightforward methods to separate fibers which are

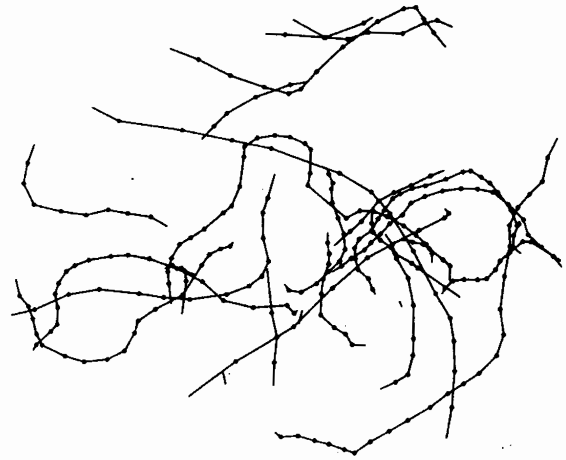

Figure 2: Tracing of Photomicrograph in Which Manual Tracing Points are Marked

otherwise overlapping. Thus the original image of Figure 2 can be transformed and rescaled into the image of Figure 3 in which individual fibers have been translated so as to eliminate overlap and to make it possible to perform measurements on isolated fibers.

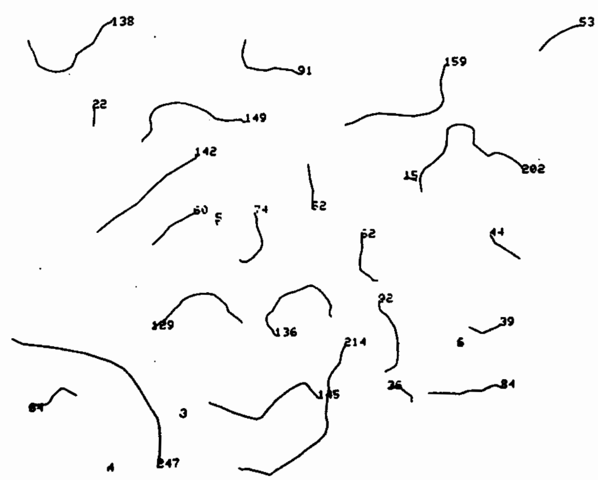

Figure 3: Image of Figure 2 in Which Individual Fibers Have Been Separated by the Computer With Lengths (in Units of 0.01 mm) Calculated for Each Fiber

B. Fiber Length Calculation

Once a sequence of points along the length of a fiber have been located, a number of possible measurements can be made. The most useful and also simplest is the measurement of fiber length. Using the points similar to those indicated in Figure 2, a sequence of straight line vectors joining those points can serve as an approximation for the original fiber. The length of the fiber then is approximated by the sum of the lengths of the straight line vectors joining those points in sequence along the fiber. For those fibers shown in Figure 2, the length of the original fiber given in units of .01 millimeter is shown in Figure 3. Thus one of the fibers in Figure 3 is as long as 2.47 millimeters and one of them as short as 0.04 millimeters. These are true projected lengths obtained by calibrating the photographic process with a microscope reticle.

Using this method we measured fiber lengths for three different pulps. The first was a southern softwood pulp for which 425 fibers were measured from a sequence of different photomicrographs. The second was a northern softwood pulp for which 620 fibers were measured. The last was a hardwood pulp for which 1,540 fibers were measured. In all 2,585 fibers were measured. A histogram showing the distribution of fiber lengths for these three pulps is given in Figure 4 in which the class lengths are 0.1 millimeter. The longest fiber is slightly more than 3 millimeters in length. Significantly different length distributions characterize the different types of pulps. Thus, as one might expect, the hardwood pulp shows a preponderance of short fibers with the softwood pulps having broader distributions and longer fibers.

With these semiautomatic data being used to approximate the shape of the fibers, there are small systematic biases leading to underestimates in the fiber lengths. One underestimate comes from the fact that the straight line cords joining points along fibers are shorter than the arc length along the fibers. That error is reduced by taking points closer together. As can be seen in the example of Figure 2, points were taken closer where there was bending in the fibers. One would expect with a fully automatic method to use points considerably closer together than is

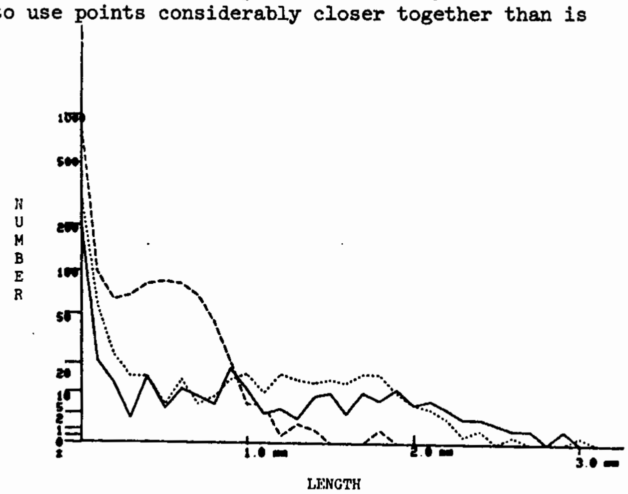

Figure 4: Histogram of Fiber Lengths for Three Pulps in Length Classes of 0.1mm. Tracings Were Made for 425 Southern Softwood Fibers (Solid Line), 620 Northern Softwood Fibers (Dotted) and 1540 Hardwood Fibers (Dashed).

practical with the semiautomatic method used here. The other systematic bias is caused by the use of fiber images projected on a plane. The effect of this error is reduced by proper microscope slide preparation.

C. Curl Measurement

By using these same fiber tracing data it is possible to make a measurement of a morphological property of the fibers that is a candidate for capturing the notion of "curl" as it is currently understood in paper physics. To understand the algorithm for curl measurement consider the image of Figure 5 in which a single fiber taken from Figure 2 is displayed enlarged. Here the tracing points are not indicated. Rather, at each internal point along the fiber, a number is shown which indicates the angle at that point between the two straight line vectors incident upon that point. Thus as we proceed around the fiber, the angles between the straight line segments constituting that fiber as it is traced are successively 10°, -26°, 33°, 8° and so on. These angles indicate the bend of the fiber at each of the points manually traced.

These angle measurements have been obtained for all of the 2,585 fibers which were traced. This produces a very large set of data and raises the question of how best these data may be understood to capture the notion of curl. One interesting possibility is indicated in Figure 6. Here we show the same image of Figure 2 with only a small subset of the tracing points being marked. Those points are marked for which the corresponding angle measurement exceeds, in this case, 45°. We thus see a set of fibers which are marked at those points which bend with an angle greater than some prescribed amount. 45°, in the case of Figure 6.

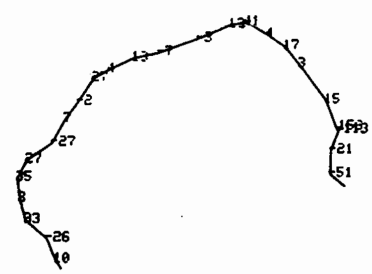

Figure 5: Single Fiber (from Center of Figure 3) With Angle of Bend Displayed at Each Plotted Point

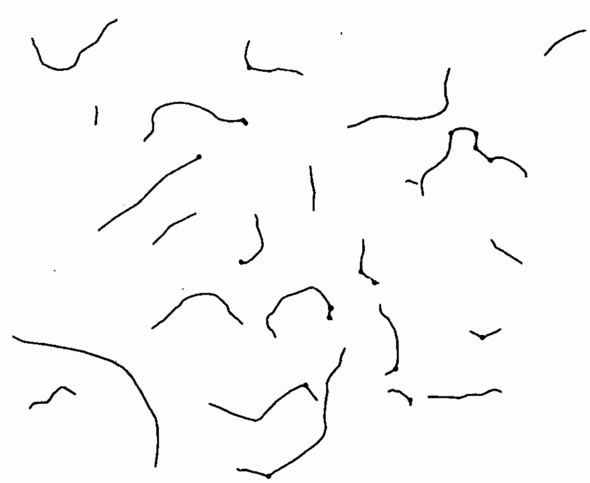

Figure 6: Image of Figure 2 With Just Those Points Marked Where Angle of Bend Exceeds 45°

One might now systematically use this method to quantify the notion of curl. Suppose we consider that fibers are divided into segments, each segment being located between two adjacent marked points or between a marked point and the end of the fiber. Thus in Figure 6 some of the segments for the 45° angle constitute the whole fiber, whereas other segments are proper subparts of a whole fiber. Some fibers are "broken" into several segments in Figure 6. We can systematically present a whole set of data corresponding to the fibers from a particular pulp by a method whose results are shown in Figures 7, 8 and 9. To understand these three dimensional surfaces, consider what would happen if we were to take the distribution of fiber segment lengths rather than whole fiber lengths as we did in Figure 4.

For each different angle and resultant segmentation as in Figure 6, we get a different histogram. Thus the histogram of fiber segment lengths corresponding to an angle of bend sharper than 180° is of course the ordinary fiber length histogram since no bend in a fiber can be greater than 180°. Thus in each of Figure 7, 8, and 9, the plane closest to the observer corresponding to an angle of 180° is merely the classical fiber length histogram, just as it appears in Figure 4. Now however, we see in addition how the histogram of fiber segment lengths changes as we change the angle of bend. In each of these three figures as one moves one's point of view away from the front plane, the shape of the distribution of fiber segments changes significantly.

Interestingly enough the change from the classical fiber length distribution to others through the use of curl information is slower for the hardwood pulp than it is for the two softwood pulps. There are many different ways to interpret these surfaces exhibiting the histogram of fiber segment lengths as a function of curl angle. Corresponding to each of these interpretations there is a different candidate measurement which captures a notion corresponding to curl. We do not presume here to choose among these various measures but rather to suggest that a choice among them may yield a candidate which can serve as a useful algorithm to quantify that notion of curl which can predict paper properties.

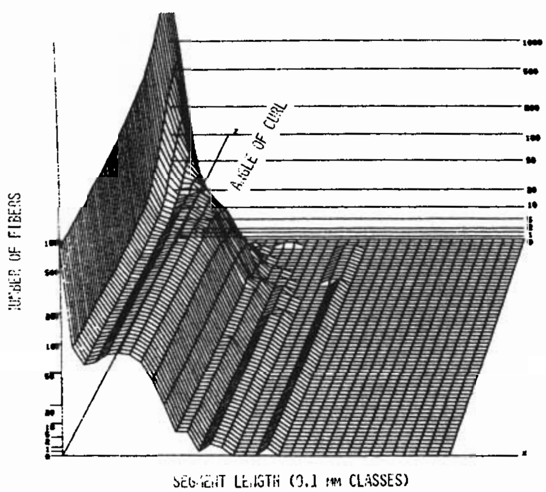

Figure 7: Histogram of Number (Y) of Fiber Segments in Length Intervals (X): 0 mm., (0.1 mm.), 3.4 mm., as Curl Angle (Z) is varied 180°, (3°), 0°. Measurements from 1540 Full Fibers Traced From Hardwood Pulp.

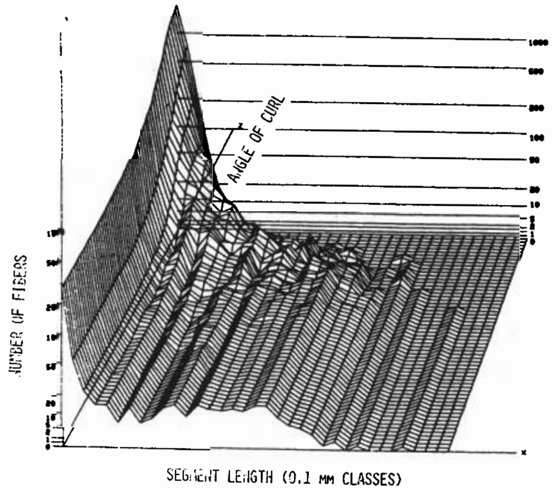

Figure 8: Histogram of Number (Y) of Fiber Segments in Length Intervals (X): 0 mm., (0.1 mm.), 3.4 mm., as Curl Angle (Z) is Varied 180°, (3°), 0°. Measurements from 620 Full Fibers Traced From Northern Softwood Pulp

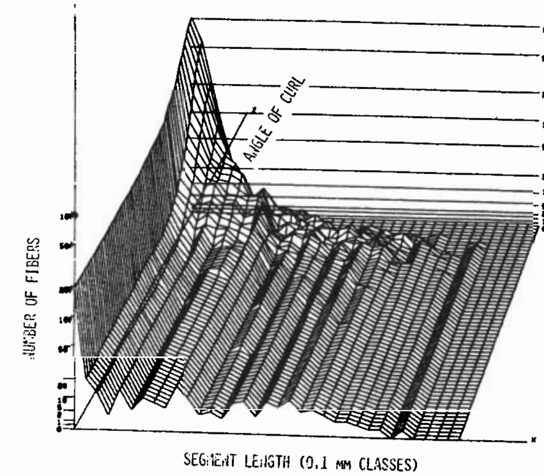

Figure 9: Histogram of Number (Y) of Fiber Segments in Length Intervals (X): 0 mm. (0.1 mm.), 3.4 mm., as Curl Angle is Varied 180°, (3°), 0°. Measurements From 425 Full Fibers Traced From Southern Softwood Pulp

D. Image Analysis Algorithms

We have attempted to establish that measurement methods based on algorithms can be applied to obtain fiber measurements which are commonly used to predict paper properties. However, to be able to claim that the measurement process has been fully automated, one must indicate how the raw data from microscopy can be captured by fully automatic means. A subsequent report will deal with these data capturing techniques and methods of fully automating them. For the present, we will exhibit some automatically acquired data and mention problems subsequently to be solved. Figure 10 shows a computer display of some automatically scanned fibers. These data were produced by scanning a photomicrograph of paper fibers similar to those shown in Figure 1. A small section of a 10.2 centimeter by 12.7 centimeter image can be displayed as in Figure 10 by using suitable printing characters to denote the different optical densities of the photographic transparency which has been scanned. With a 25 micrometer spatial resolution on the scanning device, the full resolution captured is shown in Figure 11 in which some of the internal structure of the scanned fiber can be seen. Such data can be obtained with a wide variety of scanning sensors. Ours resolves optical densities in the range from zero to 2.0 in 256 equally spaced increments. The display is produced by using characters that discriminate only a few of these density levels. To produce data such as those in Figure 2 from a scan, such as that used to produce Figures 10 and 11, requires the use of tracking algorithms.[7] The use of tracking algorithms applied to scan data will be the subject of a future report.

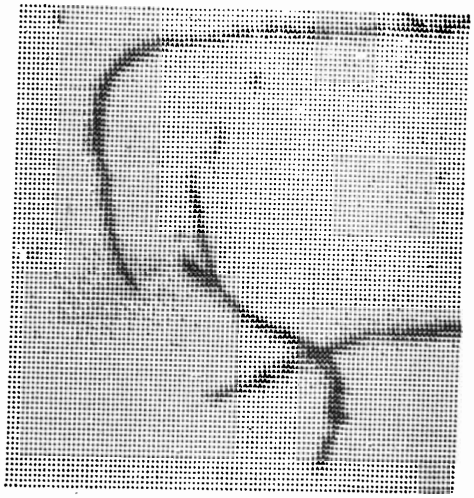

Figure 10: Computer Display of a Section of a Scanned Fiber Photomicrograph

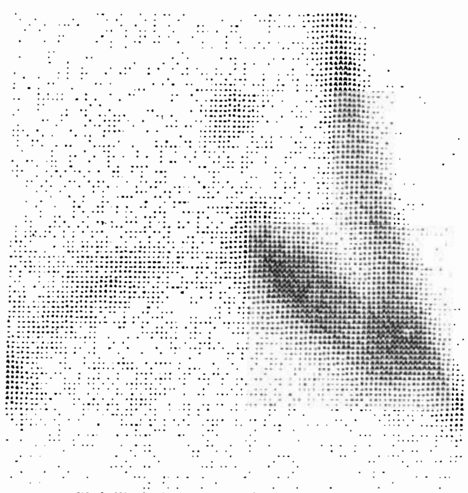

Figure 11: Display at Full 25 Micrometer Resolution of Small Section of 10.2 x 12.7 centimeter Photomicrograph Similar to Figure 1.

IV Conclusion

We have attempted to explore the class of questions concerned with making measurements on the raw materials used by the energy intensive paper industry. The measurements we have considered correspond to well established informal measurement procedures accepted by the paper industry. Although many such measurements have received some attention leading toward formalization none have yet reached the stage of formalization where they can be fully automated. By exploring the question of full automation, we hope to lead in a direction toward cooperation between the needs of the paper industry and the capabilities of the image analysis instrument industry.

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